

# A Computational Approach for Model Update of an LS-DYNA Energy Absorbing Cell

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## Abstract

NASA and its contractors are working on structural concepts for absorbing impact energy of aerospace vehicles. Recently, concepts in the form of multi-cell honeycomb-like structures designed to crush under load have been investigated for both space and aeronautics applications. Efforts to understand these concepts are progressing from tests of individual cells to tests of systems with hundreds of cells. Because of fabrication irregularities, geometry irregularities, and material properties uncertainties, the problem of reconciling analytical models, in particular LS-DYNA models, with experimental data is a challenge. A first look at the correlation results between single cell load/deflection data with LS-DYNA predictions showed problems which prompted additional work in this area. This paper describes a computational approach that uses analysis of variance, deterministic sampling techniques, response surface modeling, and genetic optimization to reconcile test with analysis results. Analysis of variance provides a screening technique for selection of critical parameters used when reconciling test with analysis. In this study, complete ignorance of the parameter distribution is assumed and, therefore, the value of any parameter within the range that is computed using the optimization procedure is considered to be equally likely. Mean values from tests are matched against LS-DYNA solutions by minimizing the square error using a genetic optimization. The paper presents the computational methodology along with results obtained using this approach.

## Introduction

The problem of updating parameters in finite element models using experimental data is one that has been studied for years. Although computer capability has increased dramatically and the size of models is perhaps at record highs, the task of

reconciling a model with test has not changed. Because the approaches used are highly diverse, several engineering societies have produced validation and verification guides Ref. [1-2] to aid users through the process. In spite of these, each problem presents unique challenges and even the application of existing approaches often require development of new metrics. From the computational viewpoint, many advances have been made to address different aspects of the problem; for example, response surface techniques to approximate solutions for computationally intensive problems, deterministic sampling approaches to improve coverage of parameter variations, efficient optimization algorithms to handle the most difficult problems, design of experiments techniques to plan and sample the design space, and of course the modeling tools to handle the most complex problems. This work uses LS-DYNA, Ref. [3], as the primary modeling tool and even though LS-DYNA offers LS-OPT [4] for design optimization, the work here uses scripts and tools developed within MATLAB, (Copyrights 1984-2007, The Mathworks Inc).

The example problem selected to demonstrate the update approach is that of a three point bend test of a single energy absorbing cell, loaded specifically to understand its behavior in shear. Of particular interest is the buckling or failure load of the cell since it relates directly to the efficiency of the honeycomb concept. Although this is a static loading problem, in this paper LS-DYNA [3] explicit solution approach is used with loads incrementally applied as a function of time until the full load is reached or the model fails. Load/displacement test results for the single cell model have been collected and the task is to recover a set of parameters to reconcile the test and the LS-DYNA model.

To update parameters using experimental data, there are several key ingredients in the mathematical formulation that must be combined to make the approach computationally efficient: the use of response surface models specifically, the Moving Least Squares (MLS) [5] and Kriging's method, [6-7], the use of a Halton-leap

deterministic sampling technique [8-9], the use of analysis of variance for parameter screening [10], and the use of a genetic algorithm for optimization [11]. Admittedly, all these are well established approaches documented in the open literature, but the combination of the various elements for parameter update is not so commonly used, and that is the main thrust of this work. Aside from the algorithms, there are other important aspects of any model update effort that must be kept in mind when choosing a solution strategy. For example, the possibility of rejecting a model because it is inadequate should always be an acceptable outcome from any solution strategy. Also important is the reliance on engineering judgment when prescribing parameter variations to ensure feasible parameter sets. All these elements have been incorporated in the solution strategy discussed in the paper.

### Description of the LS-DYNA Model

The testbed and LS-DYNA model used in this work were presented in Ref. [12] and so only a brief description is presented here for the reader to get familiar with the model. Figure 1 shows the model consisting of three major parts: the “Bakelite” stanchions, the hexagonal-shaped deformable composite cell-wall, and the flanges. The cell dimensions are cell-wall width ( $W$ ) = 0.75”, cell-wall thickness ( $t$ ) = 0.010” and overall length equal to 3.35”. The model was constructed using an element edge length of 0.05”. The Kevlar cell-wall is modeled using \*MAT\_58 or \*MAT\_LAMINATED\_COMPOSITE\_FABRIC in the LS-DYNA model. Data from tensile tests of 0°/90° coupons were used to determine approximate values of modulus and failure strain in both the longitudinal and the transverse directions. Likewise, data obtained from tensile tests of ±45° coupons were used to estimate the shear stiffness and strength. The flanges consisted of two layers of Kevlar fabric, with a total effective thickness of 0.02”. The thickness of the adhesive layer in the flanges was neglected. The specific nominal material properties used in the model to represent the Kevlar 129 fabric are shown in Table 1. The Bakelite stanchion was represented using a linear elastic material with a density of 1.356E-4 lb-s<sup>2</sup>/in<sup>4</sup>, an elastic modulus of 1.09E+6 psi, and a Poisson’s ratio of 0.25. The complete model consisted of 27,582 nodes; 16,840 hexagonal solid elements; and 8,040 Belytschko-Tsay shell elements.

The test specimen was loaded in three-point bending by applying a compressive load at the center of the hexagonal cell using a 0.25” wide bar, as shown in Fig. 1. In the test, loading was introduced with displacement control with a load rate of 2.0 ipm. The reaction points (0.25” from the bottom edge)

were assumed to be simply supported. To represent the simply supported boundary conditions, a \*BOUNDARY\_SPC\_SET was defined in the model. For this set, all of the nodes within the 0.25” by 0.75” area at the bottom of both ends of the specimen were constrained initially and later changed to a line constraint.

### Model Reconciliation Metric

To reconcile test results with analysis, a metric must be established to quantify closeness of the LS-DYNA predictions with test. Because load versus deflection data is available, for a test displacement vector  $y_i(t)$  and the LS-DYNA response for a given parameter vector  $v_i$  is  $y(t, v_i)$ , the squared error difference is,

$$J(v_i) = \sum_{k=0}^{N-1} (y(k\Delta T, v_i) - y_i(k\Delta T))^2 \quad (1)$$

where  $N$  is the number of data points and  $\Delta T$  is the elapsed time between samples. Note that this metric is written as a sum of error differences over time to highlight the fact that the explicit solution in LS-DYNA is used. On the other hand, since the load is increased at every time step, load values are proportional to time; that is, if the maximum load during a test is  $P_{\max}$  and the total simulation time (arbitrarily selected) is  $T_f = N\Delta T$ , the load increment is  $P_{\max} / N$ . This ensures that the full load is reached at the final time unless failure occurs.

With the metric defined in Eq. (1), the LS-DYNA parameters can now be adjusted to minimize the error difference over all the possible parameter sets and all experimental measurements. In this paper, a deterministic solution is sought, but if the parameter variations are prescribed in terms of distribution functions, it is straightforward to find the most likely parameter set along with its probability.

### Computational Framework

To conduct a study like this it is preferable to automate the generation of LS-DYNA solutions and parameter changes. It is also important to develop computational tools that can take advantage of models as developed by engineers. Because the impact dynamics community uses LS-DYNA routinely, the most flexible approach is to manipulate LS-DYNA input file structure directly. Figure 2 shows a data flow diagram implemented using MATLAB Script files. These script files modify the LS-DYNA keyword input file automatically to update parameter values using a priori knowledge of the parameter distributions, execute LS-DYNA, and read LS-DYNA output files. By storing the results within the MATLAB environment, all the MATLAB toolboxes are available for use.

In order to make this approach viable for computationally intensive LS-DYNA models, it is proposed (as depicted in the center of figure 2), that input-output mapping of the parameter values to LS-DYNA response outputs be captured using an adaptive response surface technique. For this task two critical elements are required; 1) an efficient response surface technique, and 2) an efficient multi-dimensional sampling technique. Comments on the selection of both approaches are provided next.

### Moving Least Squares (MLS) Response Surface Formulation

A response surface model is a mathematical representation of input variables (variables that the user controls) and output variables (dependent variables). Many papers have been published on response surface techniques and one of the approaches selected was developed by Krishnamurthy as described in Ref. [5]. In this formulation the input/output relationship is given in parametric form as

$$\begin{aligned}\hat{U} &= p^T A^{-1} B U \\ A &\triangleq \sum_{i=1}^N w_i(v) p(v_i) p_i^T(v_i) \\ B &\triangleq \sum_{i=1}^N w_i(v) p(v_i) \\ P^T &\triangleq [p(v_1) \quad p(v_2) \quad \dots \quad p(v_N)]\end{aligned}\quad (2)$$

where  $\hat{U} \in \mathbb{R}^{1 \times qN}$  is a vector of predictions,  $U \in \mathbb{R}^{1 \times qN}$  is a vector of responses (in this case LS-DYNA solutions and stacked row-wise),  $v_i$  is the  $i^{\text{th}}$  parameter vector from a sample population whereas  $v$  is a variable representing the parameters,  $N$  is the population size (also number of time steps),  $w_i(v)$  is a user-defined function that weights the proximity of other parameter vectors on the response surface,  $q$  is the number of outputs (sensors), and  $p(v)$  is a set of basis functions. Krishnamurthy provided several weighting functions to handle problems with different continuity requirements given as a function of the proximity radius, where the radius was defined as  $\rho = \|v_i - v\|_2 / l$  and  $l$  is a user defined distance. In our implementation of MLS, the proximity radius is computed directly from data using a quadratic search to minimize the error between the data and the response surface prediction. Also to avoid having a catalog of weighting functions for problems with different continuity requirements, the  $\text{sinc}(\rho) = \sin(\rho) / \rho$  function is used instead.

To assess accuracy of the response surface technique from a multidimensional viewpoint, the dot product of the response surface prediction vector and

the LS-DYNA solution vector is used to estimate the error in vector angle and magnitude. For instance, consider  $\hat{Y} \in \mathbb{R}^{q \times 1}$  to be a vector of response surface predictions for  $q$  output nodes and  $Y \in \mathbb{R}^{q \times 1}$  to be a vector of LS-DYNA predictions, the angle error is

$$E_\theta = 1 - \cos(\theta) = 1 - \frac{\hat{Y}^T Y}{|\hat{Y}| |Y|} \quad (3)$$

and the magnitude error is

$$E_m = 1 - \frac{|\hat{Y}|}{|Y|} \quad (4)$$

These two metrics are used later to compare techniques.

### Kriging's Response Surface

This alternate response surface formulation has been thoroughly studied and documented in numerous papers; two excellent references are [6] and [7]. In this approach estimates of the response are computed using

$$\hat{y}(v) = \hat{\beta} + r(v)^T R^{-1} (Y - \hat{\beta} \mathbf{1}) \quad (5)$$

with

$$\begin{aligned}\hat{\beta} &= (\mathbf{1}^T R^{-1} \mathbf{1})^{-1} \mathbf{1}^T R^{-1} Y \\ R(v_i, v_j) &= \exp[-\theta \sum_{k=1}^m (v_i^k - v_j^k)^2] \\ &\text{for } i=1,2,\dots,N \text{ and } j=1,2,\dots,N \\ r^T(v) &= [R(v, v_1) \quad R(v, v_2) \quad \dots \quad R(v, v_N)] \\ Y^T &= [y_1 \quad y_2 \quad \dots \quad y_N]\end{aligned}$$

where  $y$  and  $\hat{\beta}$  are scalars,  $R \in \mathbb{R}^{N \times N}$  is a correlation matrix,  $r(v) \in \mathbb{R}^{N \times 1}$  is a correlation vector,  $Y \in \mathbb{R}^{N \times 1}$  is a vector of responses,  $\mathbf{1} \in \mathbb{R}^{N \times 1}$  is a vector whose elements are all ones,  $\theta$  is a scalar parameter yet to be computed, and  $m$  is the number of design variables. Although Eq. (5) is written for a system with a scalar output, the formulation can be applied sequentially for cases with multiple outputs. A subtle but critical aspect of this formulation is that the unknown parameter  $\theta$ , used to define the correlation matrix  $R$ , must be computed using maximum likelihood estimation. That is  $\theta$  is the value that maximizes the scalar metric

$$L(\theta) = -\frac{1}{2} (N \ln(\sigma^2) + \ln |R|), \quad \theta \in [0, \infty] \quad (6)$$

For an estimated standard deviation

$$\sigma^2 = \frac{1}{N} (Y - \hat{\beta} \mathbf{1})^T R^{-1} (Y - \hat{\beta} \mathbf{1})$$

Solving this scalar optimization problem produces a response surface fitting error that is distributed according to a Gaussian distribution with zero error at the known solutions. In contrast to the MLS approach, computing the parameter  $\theta$  requires a scalar optimization for each output vector. Hence, if the problem at hand contains  $q$  outputs sampled  $N$  times to generate a Kriging's surface one needs  $qN$  optimizations of the metric in Eq. (6).

Although computationally intensive, the solutions can be very accurate as will be demonstrated later in the paper.

#### Deterministic Sampling of the Input Parameters

A critical step when creating response surface models is in the sampling of the parameter domain. That is, having selected a set of parameters as our inputs (control variables) to the response surface algorithm, sampling of parameter values over their prescribed domain is critical. For this purpose the Halton-leap low discrepancy deterministic sampling approach described in Ref. [8] and studied extensively in Ref. [9] has been selected. The selection is based not only on the improved convergence of statistical parameters that this approach provides (over strictly random sampling) but also in that it allows setting of the problem sequentially. For example, if the number of LS-DYNA runs required is unknown a priori, one may need to add solutions later in the process. Using Halton-leap, this procedure is simple to do without the risk of generating repeated solutions or sacrificing coverage, which is not the case with many conventional sampling techniques.

#### **Discussion of Results**

The model as developed and reported in Ref. [12] is initially used to assess the effects of parameter uncertainties in the predicted displacements. At first, engineering judgment is used to select a set of parameters that is deemed uncertain. In addition, parameters that control the execution of LS-DYNA are also included. Of course, these parameters can appear anywhere in the LS-DYNA input file but for our purposes, only those in the MAT 58 are considered. The top seven parameters in Table 1 are chosen and parameter values are generated using Halton-leap deterministic sampling. Initially, 40 LS-DYNA runs are executed to create a 2<sup>nd</sup> order response surface (admittedly only 36 solutions are needed) and to conduct analysis of variance.

#### Nominal Test and LS-DYNA comparisons

Figure 3a shows the load versus displacement variations as parameters are varied over their pre-defined ranges. The response envelope; test results upper bound, lower bound, and mean values from five experiments are plotted versus predicted upper, lower, and mean values from LS-DYNA; for a cell model with bottom edge area constraint. Bounds for the analysis data are computed from results after arbitrarily varying all parameters over their domain. After examining Fig. 3a it is apparent that the LS-DYNA response envelop, for arbitrary parameter variations, does not bound the measured values. This

prompted another look at the model that resulted in changes to the end constraint from an area to a line constraint. Because the initial stiffness is also over predicted, parameter 8 is added to allow for variations in the longitudinal and transverse modulus. Figure 3b shows the LS-DYNA predictions after these changes were made showing that now a large portion of the test data envelop is bounded by the LS-DYNA predictions.

#### Analysis of variance

In studies like this, the first step is always parameter importance screening, which is often conducted using analysis of variance. For this, the conventional approach is to use the design of experiment (DOE) methodology [10] to prescribe variations of the controlled variables (model parameters) and to compute the variance effects directly from the observations. Here, a deterministic sampling approach is used instead to prescribe parameter variations, and variable screening is conducted directly from the response surface by varying each parameter independently. Although this approach sacrifices some of the advantages of DOE, coverage of the parameter space is thought to be better for problems where the goal is to create accurate response surface models as opposed to minimize the number of experiments. Certainly, if the surface is not accurate, the variance estimate is in error, but for parameter screening purposes this approach is considered adequate. Results from the analysis of variance are shown in Figure 4 for the parameters in Table 1. The abscissa of Fig. 4a indicates parameter numbers ranging from 1 to 8 and the ordinate shows five outputs corresponding to nodes 529-533 in the LS-DYNA model (located along the area where the loads are applied). Because the variance analysis is conducted at each load increment (or time increment), Fig. 4a shows the case for 12.4 lbs first. When all the parameters are varied simultaneously the displacement at each node will vary and the maximum standard deviation  $\sigma_{\max}$  over all the parameter variations and all outputs is computed and used to normalize the data. At a particular output, the ratio of its displacement standard deviation  $\sigma$  to  $\sigma_{\max}$  is a measure of the how much displacement occurs at that output node. Figure 4a shows  $\sigma / \sigma_{\max}$  with blue squares sized according to the numerical value of the ratio (full square  $\sigma / \sigma_{\max} = 1$ ). Because the load is symmetric and all nodes deform practically the same amount, Fig. 4a shows full size squares throughout. For cases where the load is not symmetric, one would see variations in this ratio as a function of the output location. This procedure helps to highlight those nodes with maximum displacements.

Now for parameter screening, it is necessary to determine how much a particular parameter variation contributes to the total displacement variance. By

varying one parameter at a time, the proportion of the total variance contributed by variations of a particular parameter is computed and superimposed on Fig. 4a using squares with black lines. Thus, Fig. 4a shows that at a load of 12.4 lbs parameter 3 (shear modulus) and 8 (longitudinal and transverse modulus) are the two largest contributors to the displacement variance. In addition to the variance, one should also consider the effects of parameter variations on the mean response. For this case, the mean value corresponding to each output is printed to the right of Fig. 4a. Also the center of the black squares coincides with the center of the blue squares only when both mean values (that is the output mean and the mean from parameter changes), are the same.

In contrast, Fig. 4b shows the same variance information but for a load case of 274lbs. Note that the variance contribution from parameters 3 and 8 is now insignificant as compared to the contribution from parameters 4 (SLIMS) and 5 (ERODS). Moreover, parameters 1, 6, and 7 also have noticeable contributions, but from Figs. 4a and 4b, parameter 2 (strain limit) had a very small impact on the displacement variations and thus could have been removed (however, it was not removed). An important conclusion drawn from the variance analysis is that parameter sensitivity (in terms of variance) is a function of load (time) and must be computed as such or risk removing critical variables from the problem.

#### Evaluation of Response Surface Techniques

With LS-DYNA solutions at hand and response surfaces computed using both MLS and Kriging, the phase  $E_\theta$  and magnitude error metrics  $E_m$  defined in Eqs. (3-4) are easily computed at each known LS-DYNA solution. Figure 5a shows  $E_m$  using MLS and  $E_\theta$  as a function of the solution number, which ranged from 1 to 87. Likewise, on Fig. 5b are the corresponding results using Kriging's approach. Note that the Kriging solution is orders of magnitude more accurate than MLS when using this metric because it forces exact matching of LS-DYNA solutions. This accuracy is only possible after optimizing for a load/time varying solution for  $\theta$ .

#### Model Reconciliation and Optimization

After a satisfactory response surface model is initially created, the next step is to use it to solve for a parameter set that minimizes the performance metric in Eq. (1). This optimization process is iterative in nature. Figure 6 shows intermediate results for cycle 2 of the optimization process. Specifically, Fig. 6a shows the MLS predictions, the test results envelope, and LS-DYNA predictions

when using cycle 2 optimized parameters, whereas the plot in Fig. 6b shows Kriging's results. In the context of this work, a cycle (or iteration) starts by creating a response surface model from an initial set of LS-DYNA solutions, as depicted in Fig. 7, followed by an estimation of an optimum parameter set. To verify this optimum solution, LS-DYNA is executed using the optimized parameter set and this solution is compared to our response surface predictions. This comparison is shown in Fig. 6 after going through the cycle twice. If the comparison is satisfactory the optimization cycle can be stopped. Otherwise, this suboptimal LS-DYNA solution is added to the LS-DYNA solution set and an updated response surface model is created. At this point, the cycle starts again until convergence is achieved. Naturally, solutions after each cycle with MLS and Kriging are different because the response surface models are different. For example, Table 2 shows the optimized parameter set obtained using MLS and Kriging's with 155 LS-DYNA solutions. Figure 8 shows the results using Kriging's method after completing 3 cycles. The plot in figure 8a shows mean values for Kriging, LS-DYNA, and the mean of the test results. The plot shown in figure 8b includes the same data but the test results are shown in terms of bounds. Because Kriging and LS-DYNA predictions are close to the mean test results, the optimization cycle was stopped. Conversely, the MLS optimization did not converge after 4 cycles and so results are not shown.

#### **Concluding Remarks**

A methodology to update LS-DYNA generated models has been successfully demonstrated that uses response surface techniques to reduce computational burden, analysis of variance for parameter screening, a genetic optimization algorithm for parameter updates, and deterministic sampling for efficient sampling of the parameter space. To illustrate the approach, an LS-DYNA model of an energy absorbing composite cell is used and 8 parameters were updated based on experimental data. In this example, load/displacement data from five experiments are used to recover a parameter set to reconcile test with analysis. Initial analysis of variance was instrumental in understanding deficiencies in the modeling approach and pointed to improvements that resulted in better correlation even without optimization. Results show that the prediction error is significantly reduced and in the end, the LS-DYNA model accurately predicted the mean response measured experimentally.

Because complete ignorance of the parameter uncertainties is assumed, it is not possible to assess the probability that this optimized parameter set is correct. Nonetheless, the computational framework can incorporate distribution functions that would allow users to estimate this probability as well.

Finally, of the two different response surface techniques, Moving Least Squares (MLS) and Kriging, results using Kriging's approach proved to be more accurate than MLS but the computational burden is significantly higher.

### References

1. Anonymous: "Guide for the Verification and Validation of Computational Fluid Dynamics Simulations." AIAA G-077-1998, American Institute of Aeronautics and Astronautics, Reston, VA.
2. Anonymous: "ASME, V & V 10-2006 Guide for Verification and Validation in Computational Solid Mechanics", <http://catalog.asme.org>
3. Hallquist, J.O.: "LS-DYNA Keyword User's Manual," Version 971, Livermore Software Technology Company, Livermore, CA, August 2006.
4. Stander, N, Eggleston, T, Craig, K, Roux, W: "LS-OPT User's Manual A Design Optimization and Probabilistic Analysis Tool for the Engineering Analyst." Version 2, Oct. 2003.
5. Krishnamurthy, T. and Romero, V.J.: "Construction of Response Surface with Higher Order Continuity and its Application to Reliability Engineering." Proceedings of the 43rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, April 22-25, 2002, Denver, CO, AIAA Paper No. 2002-1466.
6. Sacks, J., Schiller, S.B, and Welch, W.J.: "Design of Computer Experiments," Technometrics, February 1989, Vol. 31, No. 1
7. Krishnamurthy, T.: "Comparison of Response Surface Construction Methods for Derivative Estimation Using Least Squares, Kriging, and Radial Basis Functions," Proceedings of the 46<sup>th</sup> AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, 18-21 April, 2005, Austin TX, AIAA-2005-1821.
8. Halton, J.H., "On the Efficiency of Certain Quasi-Random Sequences of Points in Evaluating Multi-Dimensional Integrals," Numerische Mathematik, Vol. 2, pp. 84-90, 1960.
9. Robinson, D., Atcitty, C., "Comparison of Quasi- and Pseudo-Monte Carlo Sampling for Reliability and Uncertainty Analysis," American Institute of Aeronautics and Astronautics, A99-24882, and AIAA-99-1589.
10. Montgomery, D.C.: Design and Analysis of Experiments, 6<sup>th</sup> edition, Copyright 2005, John Wiley & Sons, Inc.
11. Chipperfield, A., Fleming, P. J., Pohlheim, H. and Fonseca, C. M.: "Genetic Algorithm Toolbox for use with Matlab." Technical Report No. 512, Department of Automatic Control and Systems Engineering, University of Sheffield, 1994.
12. Kellas, S. and Jackson, K.E., "Deployable System for Crash-Load Attenuation," Proceedings of the American Helicopter Society (AHS) 63<sup>rd</sup> Annual Forum, Virginia Beach, VA, May 1-3, 2007.

**Table 1. MAT 58 Material Properties and distribution values**

<b>Parameter No.</b>	<b>Material Property</b>	<b>Upper Bound</b>	<b>Lower Bound</b>	<b>Nominal</b>
1	Stress limit of nonlinear portion of shear curve psi	8000	2000	4156.3
2	Strain limit of nonlinear portion of shear curve in/in	0.0374	0.00935	0.022509
3	Shear modulus, psi	614,962.0	1.5374e+5	4.563e+5
4	SLIMS	0.75	0.1	0.30087
5	ERODS	0.8	0.05	0.28554
6	Strain at shear strength, in/in	0.1	0.01	0.053669
7	Shear strength, psi	14000.0	3750	8077
8	Young's modulus, longitudinal and transverse, psi	1.6E+6	1.3e+006	1.3183e+6

**Table 2. Optimized parameter values using MLS and Kriging**

<b>Parameter No.</b>	<b>Material Property</b>	<b>Nominal</b>	<b>Optimum MLS (155)</b>	<b>Optimum KRG (155)</b>
1	Stress limit of nonlinear portion of shear curve psi	4156.3	8000	7020.8
2	Strain limit of nonlinear portion of shear curve in/in	0.022509	0.034969	0.00935
3	Shear modulus, psi	4.563e+5	1.5374e+5	1.5374e+5
4	SLIMS	0.30087	0.64059	0.1
5	ERODS	0.28554	0.05	0.73279
6	Strain at shear strength, in/in	0.053669	0.041606	0.094463
7	Shear strength, psi	8077	3750	12383
8	Young's modulus longitudinal direction and transverse, psi	1.3183e+6	1.3e+006	1.3e+006

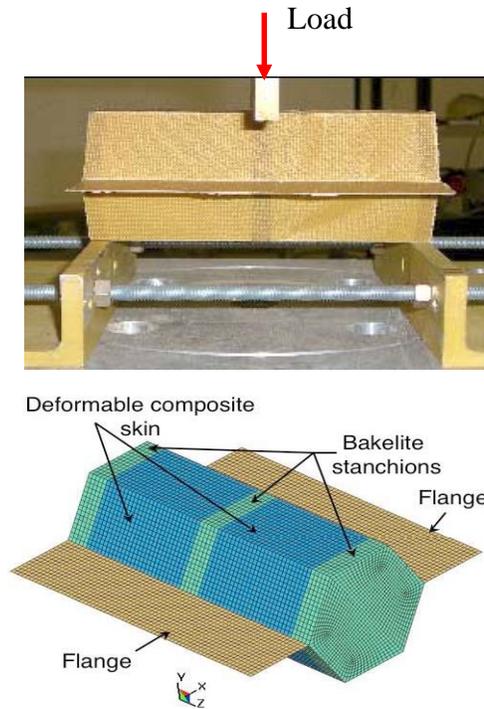


Fig. 1 Picture of cell testbed and LS-DYNA model

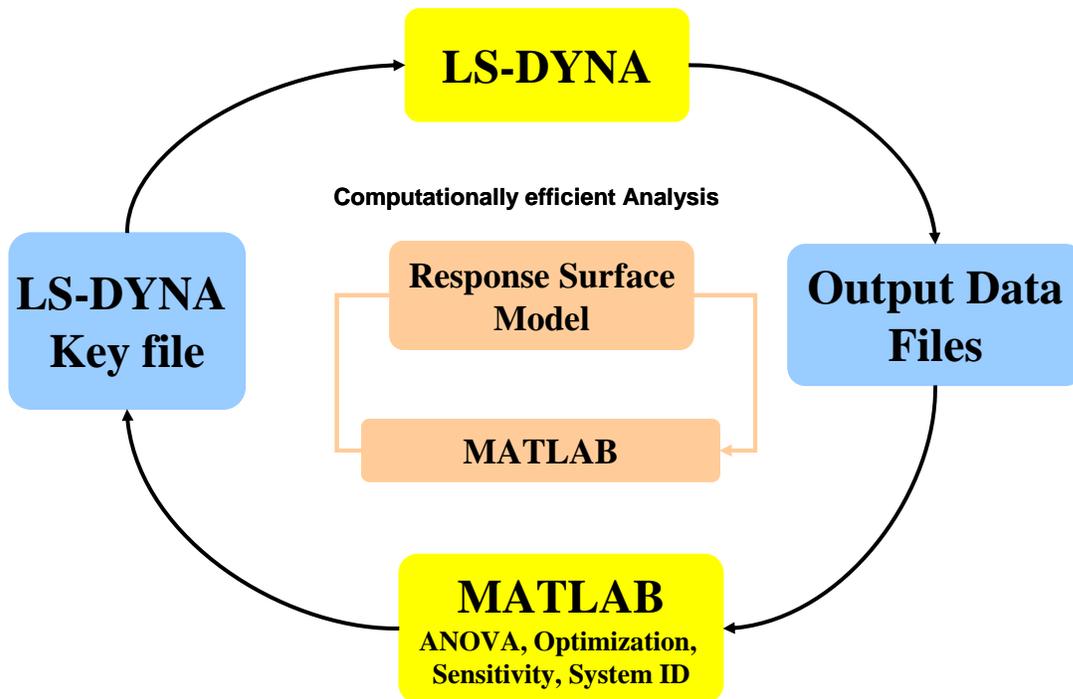


Fig. 2 Computational framework for update problem

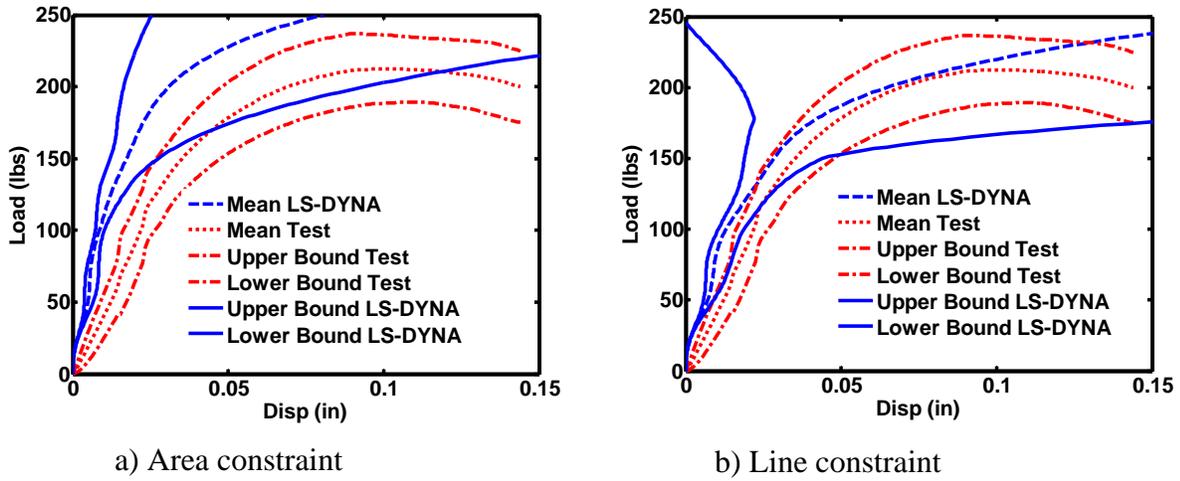
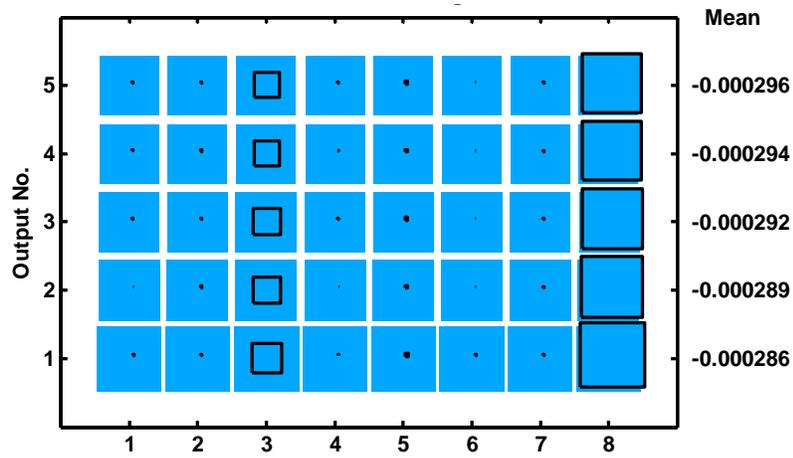
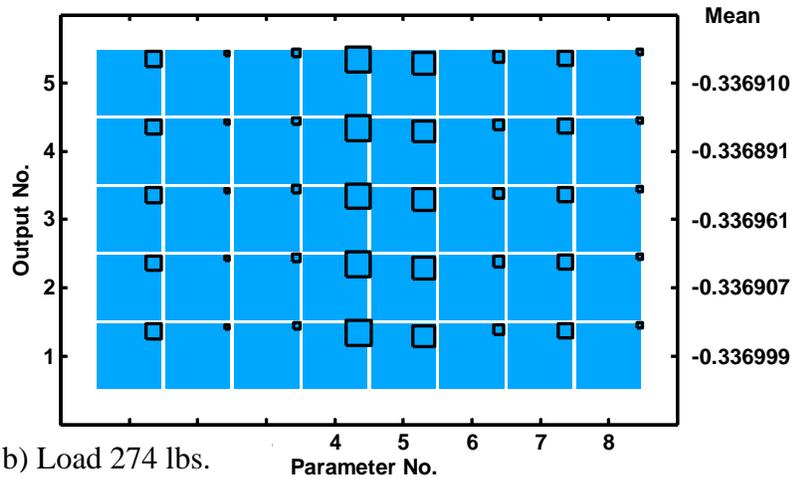


Fig. 3. Comparison of original LS-DYNA model (3a) with area constraint versus model with line constraint (3b)



a) Load 12.4 lbs.



b) Load 274 lbs.

Fig. 4 Analysis of variance at two different loads for outputs 1-5 corresponding to displacements in the y direction for nodes 529-533

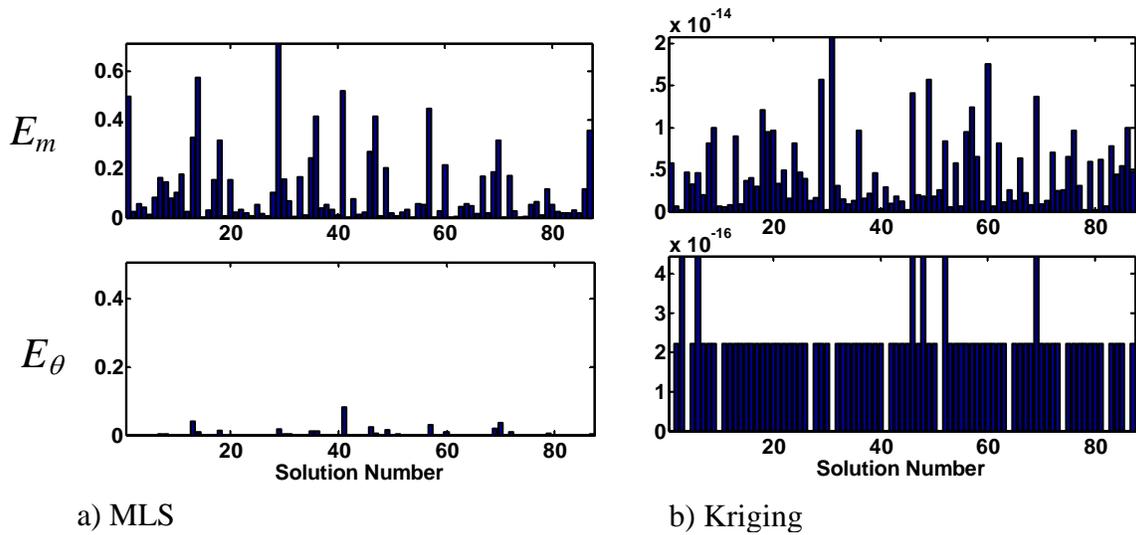


Fig. 5. Comparison of displacement prediction error with MLS and Kriging.

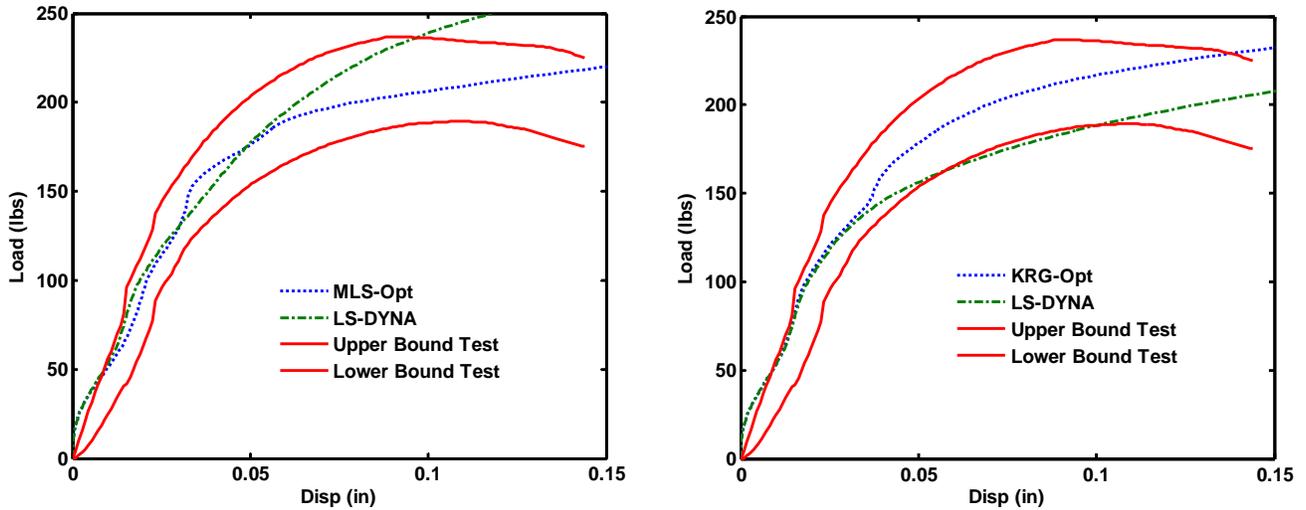


Fig. 6 Comparison of test results against MLS and Kriging's response surface procedure with the optimized LS-DYNA solution (cycle 2)

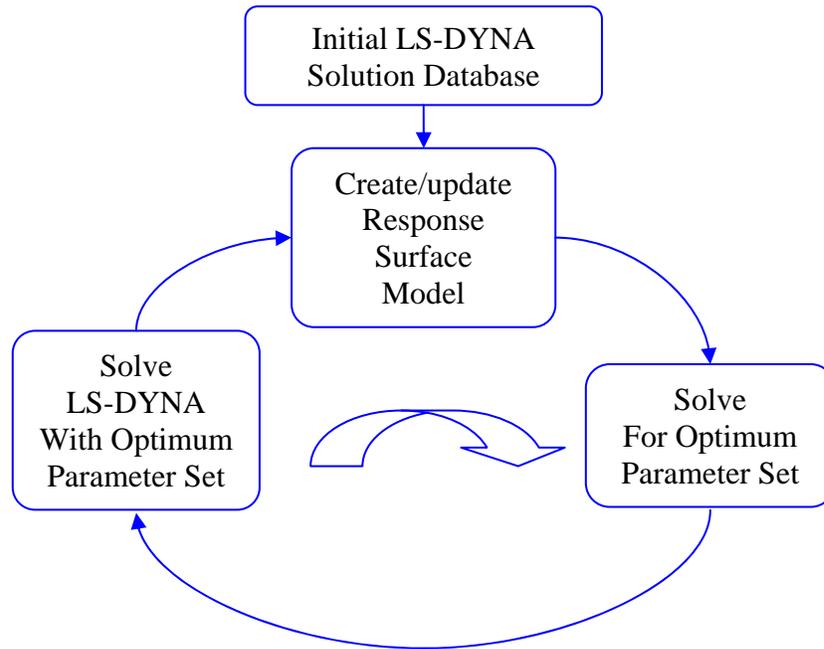


Fig. 7 Optimization design cycle

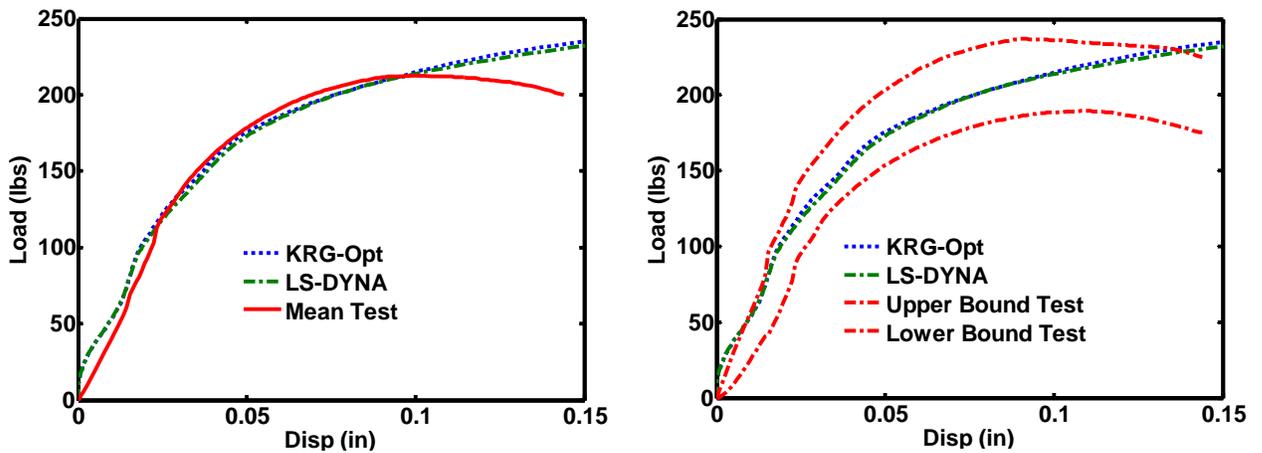


Fig. 8 Comparison of test mean and bounds against Kriging's response surface model and LS-DYNA using the optimized parameters (cycle 3)